ON THE STRENGTH OF TELEGRAPH WIRES.

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1. The use of the flexible cable or wire as a constructive element suggests a variety of problems possessing more or less interest both for the geometer and for the engineer. The following note is an attempt to treat those problems which arise in the comparatively little studied case of the suspended wires used for telegraph lines, telephone lines, and the like.

2. We assume that the wire is perfectly flexible, homogeneous, and of uniform diameter. These assumptions are so near to the truth that the results deduced from them may be applied to the solution of constructive problems. The forces which act on the wire are its weight and the pressure of the wind blowing against it. The former is estimated at 480 pounds to the cubic foot; the latter at 75 per cent. of the pressure on the elevation of the wire. The joint load therefore in the most unfavorable case, where the two forces coincide in direction, will be in tons per inch of length

$$q = \frac{79W + 222PW^{\frac{1}{2}}}{10^{10}}$$

where W is the weight of the wire in pounds per mile of length, and P the greatest wind pressure in pounds per square foot.

The greatest tension admissible in such a wire is in tons

$$T = \frac{\sigma W}{17600}$$

where σ is the modulus of tenacity in tons per square inch.

3. The general differential equation of the catenary of such a wire is

$$dy = \frac{Q}{H} \cdot dx$$

where Q denotes the total load applied to the arc measured from the lowest point to (x, y) and H the tension in the wire at that lowest point. Two cases must be distinguished. In the first, the catenary is so flat that the load carried by the wire may be treated as uniformly distributed over the span. In the second, the curvature is sharper, and the load is treated as uniformly distributed over the arc.

I. LOAD UNIFORM PER INCH OF SPAN.

4. In this case $Q = qx$, where q is constant and the catenary

$$y = \frac{qx^2}{2H}$$

is the common parabola. The length of the arc may be expressed in finite terms by elementary transcendents. For actual computations, however, it is more convenient to develop dl into a series, thus: —

$$\frac{dl}{dx} = 1 + \frac{1}{2}\left(\frac{qx}{H}\right)^2 - \frac{1}{8}\left(\frac{qx}{H}\right)^4 + \frac{1}{16}\left(\frac{qx}{H}\right)^6 - \dots;$$

whence for the length l of the arc measured from the lowest point (0,0) to (x, y), we get if $y = \rho x$

$$\frac{l}{x} = 1 + \frac{2}{3}\rho^2 - \frac{2}{5}\rho^4 + \frac{4}{7}\rho^6 - \dots.$$

In all problems in which ρ^4 is negligible this approximate formula furnishes results practically coincident with those of the more exact formula given below. The principal questions which arise in practice are as follows: —

5. We put s for span, f for dip of wire, and ρ for the ratio $2f : s$. Then the horizontal tension H in the wire and the sloping tension T are respectively

$$H = \frac{qs}{4\rho}, \qquad T = \sqrt{\left\{H^2 + \left(\frac{qs}{2}\right)^2\right\}}.$$

The following sub-cases are noted: —

1′. Given span, dip, wind-pressure, weight, and strength of wire; to find the stress therein.

$$\sigma = 8800\, s\sqrt{\left(1 + \frac{1}{4\rho^2}\right)} \cdot \frac{q}{W}.$$

$$\frac{q}{W} = \frac{79 + 222\, PW^{-\frac{1}{2}}}{10^{10}}.$$

2′. Given span, wind-pressure, weight, and strength of wire; to find the dip.

$$\frac{1}{4\rho^2} = \left(\frac{2}{s} \cdot \frac{T}{q}\right)^2 - 1,$$

$$\frac{T}{q} = \frac{\sigma}{17600} : \frac{q}{W},$$

and $\frac{q}{W}$ is found as above.

3′. Given dip, wind-pressure, weight, and strength of wire; to find the span.

$$\frac{1}{\rho^2} = \sqrt{\left\{1 + \left(\frac{2}{f} \cdot \frac{T}{q}\right)^2\right\}} - 1,$$

and $\frac{T}{q}$ is found as above.

4'. Given span, dip, wind-pressure, and strength of wire; to find its weight.

$$\sqrt{W} = \frac{222\,P}{\dfrac{10^{10}}{\left(\dfrac{W}{q}\right)} - 79},$$

$$\frac{W}{q} = \frac{8800\,s}{\sigma}\sqrt{\left\{1 + \frac{1}{4\rho^2}\right\}}.$$

Thus, for example, in a wire admitting a working stress of 12 tons on the square inch, if the span is 200^f, the dip 30^i, and the greatest anticipated wind-pressure 30 pounds on the square foot,

$$\rho = \frac{1}{40},$$

$$\frac{W}{q} = 1760000\sqrt{401},$$

$$\frac{10^{10}}{\left(\dfrac{W}{q}\right)} = 283.7,$$

$$\sqrt{W} = 31.76,$$

and the necessary weight of wire for the line is 1009 pounds to the mile

II. LOAD UNIFORM PER INCH OF ARC.

6. In this case $Q = qs$ and the differential equation to the catenary is

$$dy = \frac{q}{H}sdx.$$

This is the familiar case of the common catenary. The equation in integral form is, with the origin at the lowest point,

$$y = \frac{H}{2q}\left(e^{\frac{qx}{H}} + e^{-\frac{qx}{H}}\right) - \frac{H}{q},$$

and the length of the arc from this origin to (x, y) is

$$l = \frac{H}{2q}\left(e^{\frac{qx}{H}} - e^{-\frac{qx}{H}}\right).$$

These exponential forms are for more convenient use in practical applications developed into infinite series. The following are the results which furnish values of x, y, and l in terms of $\rho = y : x$:—

$$x = \frac{2H}{q}(\rho - \frac{1}{3}\rho^3 + \frac{13}{45}\rho^5 - \ldots),$$

$$y = \frac{2H}{q}(\rho^2 - \frac{1}{3}\rho^4 + \frac{13}{45}\rho^6 - \ldots),$$
$$\frac{l}{x} = 1 + \frac{2}{3}\rho^2 - \frac{14}{45}\rho^4 + \frac{278}{945}\rho^6 - \ldots.$$

These more exact formulæ should be used in all cases in which the fourth or higher powers of ρ cannot be neglected; as, for example, in designing great river spans and the like. The reduction of the results to forms suitable for computation will now be given. The series have been found by actual trial to furnish results with greater accuracy and facility than the tables of hyperbolic sines and cosines.

7. We put as before s for span, f for dip, and ρ for the ratio $2f:s$. Then

$$T = H + qf,$$

and

$$H = \frac{qs}{4\rho}(1 + \frac{1}{3}\rho^2 - \frac{8}{45}\rho^4 + \ldots).$$

The principal sub-cases are noted.

1′. Given span, dip, wind-pressure, and weight of wire; to find the stress therein.

$$\sigma = \frac{17600}{W} \cdot \frac{T}{q} \cdot q,$$
$$\frac{T}{q} = \frac{H}{q} + f,$$
$$\frac{H}{q} = \frac{s}{4\rho}(1 + \frac{1}{3}\rho^2 - \frac{8}{45}\rho^4 + \ldots),$$

and q is found from formula in Art. 2.

2′. Given dip, wind-pressure, weight, and strength of wire; to find the span. We compute q and T from formulæ in Art. 2; then

$$H = T - qf,$$

and s is found from the transcendental equation

$$e^{\frac{qs}{2H}} = \sqrt{\frac{T+H}{2H}} + \sqrt{\frac{T-H}{2H}}$$

by the aid of a table of Naperian logarithms.

3′. Given span, wind-pressure, weight and strength of wire; to find the dip. We compute q and T from formulæ in Art. 2, and $\frac{q}{H}$ from the transcendental equation

$$\frac{qs}{2H} = \frac{qs}{4T}\left(e^{\frac{qs}{2H}} + e^{-\frac{qs}{2H}}\right) = \frac{qs}{2T}\operatorname{coh}\frac{qs}{2H}.$$

Then

$$f = \frac{T}{q} - \frac{H}{q}.$$

As the solution of such equations by trial and error is a tedious process, we express $\frac{qs}{2H} = h$ in terms of $\frac{qs}{2T} = t$ as follows: —

By developing the exponentials into series and dividing we find

$$t = h - \frac{1}{2}h^3 + \frac{5}{24}h^5 - \frac{61}{720}h^7 + \ldots.$$

Reverting the series we get

$$h = t + \frac{1}{2}t^3 + \frac{13}{24}t^5 + \frac{541}{720}t^7 + \ldots.$$

Having computed t from the original data we find h from this series and then calculate

$$f = \frac{s}{2t} - \frac{s}{2h}.$$

4′. Given span, dip, wind-pressure, and strength of wire; to find its weight. It is easy to show that

$$\frac{T}{q} = \frac{s}{4\rho}\left(1 + \frac{7}{3}\rho^2 - \frac{8}{45}\rho^4 + \ldots\right),$$

whence

$$\frac{W}{q} = \frac{17600}{\sigma}\cdot\frac{T}{q}$$

and

$$\sqrt{W} = \frac{222P}{\dfrac{10^{10}}{\left(\dfrac{W}{q}\right)} - 79}$$

For example, a river crossing of 2000′ constructed of steel wire weighing 550 pounds to the mile, and possessing a working strength of 30 tons to the square inch, if exposed to a wind pressure of 40 pounds to the square foot should have a dip computed as follows. We find

$$q = 0.00002517,$$
$$T = 0.9375;$$

$$\therefore t = 0.3222,$$
$$h = 0.3411\,;$$

whence

$$f = \frac{1000}{t} - \frac{1000}{h} = 172^f.$$

8. It is sufficiently obvious that the dip corresponding to the greatest admissible stress is that which the wire must have when the temperature is at the lowest. The line will ordinarily be erected at a higher temperature and the necessary increment must be given to the dip. To determine this increment we compute the length of wire l in the whole span from the formula

$$\frac{l}{s} = 1 + \frac{2}{3}\rho^2 - \frac{14}{45}\rho^4 + \frac{278}{945}\rho^6 - \ldots,$$

and then calculate the increased length L at the higher temperature, using the appropriate co-efficient of dilation; which for iron or steel wire is about 1:150,000 for each degree Fahrenheit. Then from the same formula we compute the value P of ρ corresponding to the increased value L of l, and thence the increased value F of the dip. For convenience we revert this series also. Putting λ for $\frac{l}{s} - 1$ we find

$$\rho^2 = \frac{3}{2}\lambda + \frac{21}{20}\lambda^2 - \frac{27}{1400}\lambda^3 + \ldots.$$

For example, in the case just treated, if the lowest temperature be $-20°$ F. and the temperature on the day of erection be $55°$ F., we find

$$\rho = 0.1720\,; \qquad \therefore \frac{l}{s} = 1.0194.$$

Increasing this by 1 : 20,000 we get

$$\frac{L}{s} = 1.0199\,;$$
$$\therefore \lambda = 0.0199, \qquad \rho^2 = 0.0303, \qquad \rho = 0.174,$$

and the increased dip is 174^f, the length of cable at the same time being 2040^f, An apparent increase of precision in such results can be readily attained by the more liberal use of decimal places. But the uncertainty of the data, particularly of the wind-pressure, is so great that no confidence should be bestowed on the results of such calculations. It may be remarked that in general the data in engigineering problems can themselves be relied on only to three significant figures and the results of computations based on them can of course possess no higher degree of precision.